

List of Topics for the PhD Qualifying Exam

Topics in Probability

Some Basic Concepts:

Combinatorial Probability, Discrete/Continuous Random Variables, Expectation/Variance, Common Probability Distributions, Joint/Marginal Distributions, Conditional Distributions and Independence, Characteristic Functions.

Measure Theoretic Foundations:

σ -algebras, Probability Measures and Distributions, Borel-Cantelli Lemmas, 0-1 Laws. Conditional Probability and Expectation (measure theoretic view), Martingales.

Limit theorems:

Various Modes of Convergence, Central Limit Theorem, Laws of Large Numbers.

Some Stochastic Processes:

Poisson Processes, Markov Chains (Classification of States/Stationary Distribution), Random Walks, Brownian Motion, Renewal Processes.

References

1. Billingsley, Probability and Measure
2. Chung, A Course in Probability Theory

Topics in Statistics

Properties of Random Samples:

Sampling from the Normal Distribution, Exponential Families, Location-Scale Families, Order Statistics, The Delta Method.

Principles of Data Reduction:

Sufficiency, Minimal Sufficiency, Ancillary and Complete Statistics, Basu's Theorem.

Point Estimation:

Method of Moments, Maximum Likelihood Estimators, Bayes Estimators, Mean Squared Error, Best Unbiased Estimators, Fisher information, Cramer-Rao Lower Bound, Rao-Blackwell Theorem, Loss Function Optimality

Interval Estimation:

Inverting a Test, Pivotal Quantities, Bayesian Intervals, Size and Coverage Probability, Test-Related Optimality, Bayesian Optimality.

Hypothesis Testing:

Neyman-Pearson Lemma, Likelihood Ratio Tests and Its Asymptotic Theory, Bayesian Tests, Union-Intersection and Intersection-Union Tests, Power Function, Most Powerfull Test, p-Values.

Linear Models:

Random Vectors and Matrices, Multivariate Normal Distributions, Distributions of Quadratic Forms, Least Squares Estimation Theory, Gauss-Markov theorem, Theory of Testing Hypotheses, ANOVA and Regression.

References

1. *Theory of Point Estimation* by E.L. Lehmann and George Casella.
2. *Testing Statistical Hypotheses* by E.L. Lehmann and Joseph P. Romano.
3. *Statistical Inference* by Casella and Berger.
4. *Plane Answers to Complex Questions: The Theory of Linear Models* by Ronald Christensen.