Complex Analysis Qualifying Exam Syllabus

Department of Mathematics, University of North Texas

1. The Complex Number System: Algebraic properties, topological and metric properties, polar representations and powers of complex numbers, the extended plane and stereographic projection.

2. Power Series: Formal and convergent power series, operations with power series, analytic functions as power series, differentiation and integration of power series, elementary functions (exponential function, logarithm, trigonometric functions).

3. Analytic Functions: Complex differentiation, integrals over paths, Cauchy-Riemann equations, Cauchy's integral formula, Morera's theorem, Goursat's theorem.

4. Applications of Cauchy's Integral Formula: Liouville's theorem, Laurent series, zeros, poles, residues, classification of singularities, the open mapping theorem, Rouché's theorem, maximum modulus principle, Schwarz lemma, evaluation of definite and improper real integrals.

5. Entire and Meromorphic Functions: Analytic continuation, Schwarz reflection principle, infinite products, Weierstrass products, meromorphic functions, the Γ-function, Riemann's ζ-function, the basic theory of elliptic functions, the little and great Picard theorems.

6. Conformal Mappings: Mapping by elementary functions, Möbius transformations, analytic automorphisms of the disk, upper half plane and Riemann sphere, conformal (non) equivalence of annuli, normal families, Riemann mapping theorem.

7. Harmonic Functions: Basic properties, examples and association with analytic functions, the mean value property, harmonic functions on the disk and Poisson integrals, Harnack's inequalities, the Poisson-Jensen formula.

Suggested Reading:

- L. Ahlfors, Complex Analysis
- J. Conway, Functions of One Complex Variable
- S. Lang, Complex Analysis
- J. Noguchi, Introduction to Complex Analysis

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